Evaluation of Marketing-Pricing Decisions in a Two-Echelon Supply Chain

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This paper discusses the interaction between one manufacturer and a single retailer in a channel in which both are willing to optimize their profits by advertising and pricing decisions. The manufacturer produces and sells a product at wholesale price to the retailer who in turn distributes it to consumers with retail price. The market demand is simultaneously affected by retail price, brand advertising of the manufacturer, and local advertising of the retailer. A Cobb-Douglas demand function is used to demonstrate the relationship between the parameters. Decision variables are two firms’ prices and their advertising investments. The problem is modelled under integrated policy and Stackelberg game. Also, we examine Retail Fixed Markup (RFM) policy and investigate its performance on supply chain. Then, the solution under three policies compared by numerical study, and the Pareto-efficient strategy is derived. We found numerically that a properly designed RFM policy improves each member’s profit and leads to Pareto improvement over Stackelberg policy. Besides, it improves the total supply chain’s profit by 600% in average comparing to Stackelberg policy.

Keywords: marketing, pricing, price-dependent demand, retail fixed markup, supply chain.

JEL Classification: L06, L11, M03, M31, M37, R32

Introduction

In recent years a considerable amount of research has been conducted on different aspects of supply chain such as pricing, marketing, production, distribution, purchasing, inventory management, etc. The retailer–manufacturer interaction problem is one of the classical research areas in the supply chain literature. (a) How should the members of supply chain behave in order to manage their costs/profits? (b) How can the manufacturer take advantages of his leadership in order to increase his profit and the retailer’s? Answering these questions is our purpose in this research.

To answer question (a), one approach is known as centralized/integrated policy. It ignores the competition between the members with the aim of maximizing the total system profit. But it may be impractical and undesirable in many cases due to incentive conflicts. Another approach is decentralized policy, in which each member independently chooses his strategy in a way that the overall system efficiency couldn’t necessarily be optimized. In other words, this approach often causes lost profits for the whole system comparing to centralized approach.

To answer question (b), we know that different decentralized approaches leads to different system efficiency. In the other hand, each member prefers an approach which cause more profit to him. In this research we examine two approaches: Stackelberg and RFM. In Stackelberg policy, the leader of the game (manufacturer) sets his wholesale price and national advertising investment first, and then the follower (retailer) observes this decision and chooses his retail price and local advertising. While in RFM policy, the manufacturer chooses his decisions similar to Stackelberg, but the retailer only decides on local investment. Since, he receives a fixed markup; and, choosing the wholesale price by manufacturer is equivalent to setting the retail price. It is shown that RFM policy not only improves the total channel’s profit, but also, improves each member’s profit. So, RFM policy leads to Pareto improvement over Stackelberg policy.

In this paper, we consider a two-echelon supply chain including one manufacturer in the upstream and a single retailer in the downstream of the channel. Optimal advertising investments and pricing decision is discussed in this supply chain. The manufacturer and retailer want to maximize their profit by adjusting Marketing and Pricing decisions. The market demand is simultaneously affected by retail price, and advertising efforts, and, is a decreasing and convex function of the price, but an increasing and concave function of the retailer’s and manufacturer’s advertising investment. Both firms use advertising.
programs to encourage customers to purchase the product. The manufacturer promotes the product by brand advertising, while the retailer supports the manufacturer’s product by local advertisement. We model the problem under integrated policy, and as a Stackelberg game in Bresnahan and Reiss, 1985; Kadiyali er and a single retailer channel is -ual total $\geq$, $0 c^2$ Esfahani svertising efforts of wms’ ret (2009) and Seyed w- Yang and Zhou, 2006; Chu appling Stackelberg game Srinivasan, 1987; echelon supply chain S relation to marketing strategies such as pricing policies. In the different aspects of supply chain. We don’t claim that RFM can coordinate the channel. In contrast, our results show that RFM does not coordinate the channel. We say one policy can coordinate the channel if it improves all the members’ profit comparing to other possible decentralized policies, and leads to equal total profit comparing to centralized/integrated policy. This supply chain can be considered either “Competitive” or “Cooperative”. Since, each member of the channel is willing to optimize his own decisions and objectives. In the other hand, the both members contribute to the advertising cost in order to increase the demand and their profit. The reminder of the paper is organized as follows. The next section reviews the related literature. Section 3 describes the notations, assumptions, and the two firms’ objective functions. Then we develop integrated, Stackelberg, and Retail Fixed Markup policies for the problem in section 4. We present a numerical study and the corresponding sensitivity analysis for some parameters with the purpose of evaluating the influence of these parameters on profits, the two firms’ decision and evaluating Pareto-improving region in section 5. Finally, section 6 summarizes the results and covers the concluding remarks.

**Literature review**

Many papers in the recent years have been published in the different aspects of supply chain. Our research is related, at least in spirit, to channel coordination in supply chain. Here, we review some related papers that consider marketing-pricing decisions, Retail Fixed Markup, and coordination mechanisms.

In Stackelberg game setting, Eliashberg and Steinberg (1987) consider production activities and their relation to marketing strategies such as pricing policies. Some other authors study similar problem for a two-echelon supply chain with deterministic price-dependent demand curve, either liner or Iso-elastic. (e.g. Arcelus and Srinivasan, 1987; Li et al. 1995; Ertek and Griffin, 2002). Other papers analyze multi-echelon inventory systems by appling Stackelberg game (e.g. Lau and Lau, 2004; 2005; Yang and Zhou, 2006; Chu et al. 2006; Liu et al. 2006).

There have been extensive researches that investigate the coordinating of a supply chain. Qin et al. (2007) study the problem in which the vendor offers a price discount in a system with price-sensitive demand. Cachon and Zipkin (1999) examine some incentive contracts to coordinating the two-echelon supply chain. Viswanathan and Piplani (2001) study a coordination problem in which the vendor offers a price discount to persuade the buyers to replenish only at the specific time periods. Cachon and Lariviere (2005) consider some coordination mechanisms such as revenue sharing, price-discount, buy-back, quantity discounts, franchise fee, quantity-flexibility, and sales-rebate contracts. Huang et al. (2011) study coordinating pricing, inventory decisions, and supplier selection in a supply chain in which demand is price dependent.

Some recently published papers consider coordination problem with marketing-pricing decisions in a two echelon supply chain in which the market demand simultaneously affected by price and advertising efforts of both firms: Yue et al. (2006) investigate the price discount scheme in order to achieve coordinating the channel. Karray and Martín-Herrán (2008) study a three-stage game-theoretic model; they proceed to study advertising and pricing competition between national and store brands in Karray and Martín-Herrán (2009). He et al. (2009) model this problem as a stochastic Stackelberg differential game; Szmerekovsky and Zhang (2009) model it as a Stackelberg game in which the manufacturer is the leader. Xie and Wei (2009) consider cooperative and Stackelberg game. Xie and Neyret (2009) and SeyEdEsfahani et al. (2011) investigate this problem with different demand functions by applying four game-theoretic models including cooperative, Nash, Stackelberg-retailer and Stackelberg-manufacturer games; Kunter (2012) applies cost and revenue sharing mechanism to coordinate the channel.

There is sufficient evidence that RFM exists in practice. Many industries operate by using fixed Mark-ups such as gasoline dealers, grocers, and electronics industry. In addition, RFM is also considered in marketing literature. For example see (Bresnahan and Reiss, 1985; Kadiyali et al., 1996; 1999). Liu et al. (2006) consider a RFM policy and examine the behavior of a retailer and a single manufacturer in a decentralized channel under price-dependent demand. They also formulate the problem under price-only contract, and, show that RFM leads to Pareto improvement over the price-only contract. Li and Atkins (2002) introduce a model similar to Liu et al. (2006)’s, but for marketing and operation sections in a single firm. Although, RFM policy in our model and Liu et al. (2006)’s model does not achieve channel coordination, Ha (2001) propose one policy similar to RFM under price-dependent demand that is capable to coordinate the channel.

**Model formulation**

**Notation and assumptions**

In this paper, we use a notation for representing the parameters and the decision variables to model the Marketing-Pricing problem in a two-echelon supply chain. A one manufacturer and a single retailer channel is considered, in which the manufacturer sells the product to consumers through a retailer. The manufacturer has a fixed production cost ($c_m \geq 0$) per unit product and the retailer has a fixed distribution cost ($c_r \geq 0$) per unit. Also, the manufacture sells the product with wholesale price ($w > c_m$) to the retailer who in turn sells it with the retail price ($p > w + c_h$) to the customers. The manufacturer decides
on the National advertising expenditures \(m\), and wholesale price \(w\). On the other hand, the retailer decides on the local advertising investment \(r\), and retail price \(p\). The market demand is simultaneously affected by retail price, advertising investment of the retailer and manufacturer. The manufacturer’s advertising investment planned for influencing potential consumers to consider the product’s brand. However, the retailer’s one is to motivate customers’ buying behaviour.

Suppose that the total market demand is a decreasing and convex function of the price, but an increasing and concave function of the retailer’s and manufacturer’s advertising investment. We use a Cobb-Douglas demand function to demonstrate the relationship between the parameters. A Cobb-Douglas demand function represent the relationship of the market demand to the price, retailer’s and manufacturer’s advertising investments with their elasticity parameters. Note that this function is concave/convex, continuous and has a constant elasticity. It is widely used in economics, for example Goyal and Gunasekaran (1995) propose a model in which demand is a function of price and number of times that the product advertised. We will use this demand function similar to Yu et al. (2009). Assume that the demand function is specified by \(D(r, m, p) = Kr^\alpha m^\beta / p^\gamma\) where \(k\) is a positive constant characterizing the market scale, and \(p, r\) and \(m\) represent the price, retailer’s and manufacturer’s advertising investment, respectively. Besides, \(\alpha, \beta\) and \(\gamma\) stand for the elasticity of \(r, m\) and \(p\), respectively. It is necessary to assume \(\gamma > 0\) in order to guarantee the convexity of \(D(r, m, p)\) in \(p\). In addition, \(0 < \alpha < 1\) and \(0 < \beta < 1\) to ensure the concavity of demand function in \(r\) and \(m\).

Furthermore, \(\Pi^f_i\) represent the firm’s \(i\) profit function at policy \(j\) where \(i \in \{R, M, T\}\) and \(j \in \{I, S, F\}\). \(R, M,\) and \(T\) stand for the retailer, manufacturer, and total supply chain respectively. And \(I, S,\) and \(F\) correspond to Integrated, Stackelberg, and Retail Fixed Markup (RFM) policies, respectively. We consider three policies: Centralized/Integrated, Decentralized with Stackelberg game, and Retail Fixed Mark up (RFM) policy.

**Two firm’s objective function**

The objective function of each firm has two parts: one is the revenue from selling the product and another is the cost from advertising investment. In this case the manufacturer’s profit is \((w - c_m)D(r, m, p) - m\), and the retailer profit is \((p - w - c_R)D(r, m, p) - r\). Each firm is willing to optimize his profit. The decision variables are the retail price \((p)\), wholesale price \((w)\), retailer’s advertising investment \((r)\), and manufacturer’s advertising investment \((m)\).

**Policies**

**Integrated policy**

In the integrated or centralized policy, the manufacturer together with the retailer considered as a one single firm, so, the goal is maximizing the whole supply chain’s profit. The integrated profit is an upper bound for RFM and Stackelberg policies’ profit. Here, there are three decision variables to determine, and there is no wholesale price to optimize. We denote integrated solution by \((p^*, m^*, r^*)\). The objective function is the sum of two firm’s profit:

\[
\Pi^f_i = (p - c_m - c_R)D(r, m, p) - r - m;
\]

\[
D(r, m, p) = Kr^\alpha m^\beta / p^\gamma
\]

**Proposition 1**

(I) Integrated retail price and two firm’s advertising investment are:

\[
p^* = \frac{\gamma}{\gamma - 1}(c_m + c_R).
\]

\[
m^* = \frac{k\beta + \alpha - \alpha^2}{\sqrt{(c_m + c_R)^{\gamma - 1}}(\gamma - 1)^{\gamma - 1}}.
\]

\[
r^* = \frac{k\alpha + \beta - \beta^2}{\sqrt{(c_m + c_R)^{\gamma - 1}}(\gamma - 1)^{\gamma - 1}}.
\]

(II) And, Integrated profit is:

\[
\Pi^f_i = \left(1 - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right).
\]

**Proof:** by considering \(\partial D / \partial p = -\gamma D(r, m, p) / p, \partial D / \partial r = aD(r, m, p) / r\) and \(\partial D / \partial m = bD(r, m, p) / m\), the first order conditions are:

\[
\frac{\partial \Pi^f_i}{\partial p} = D(r, m, p) \left[1 - \frac{1}{p} (p - c_m - c_R)\right] = 0
\]

\[
\frac{\partial \Pi^f_i}{\partial r} = \frac{\alpha}{r} D(r, m, p)(p - c_m - c_R) - 1 = 0
\]

\[
\frac{\partial \Pi^f_i}{\partial m} = \frac{\beta}{m} D(r, m, p)(p - c_m - c_R) - 1 = 0
\]

By solving the above equations, we get the optimal solution as in part (I). The Hessian matrix is negative definite. Since \(H_{11} < 0\), \(H_{12} > 0\) and \(H_{11}(H_{22}K_{33} - H_{23}K_{32}) < 0\). So, \(\Pi^f_i\) is concave in \(p, r\), and \(m\) and have maximum:

\[
H = \left[\begin{array}{ccc}
\frac{\partial^2 \Pi^f_i}{\partial p^2} & \frac{\partial^2 \Pi^f_i}{\partial p \partial r} & \frac{\partial^2 \Pi^f_i}{\partial p \partial m} \\
\frac{\partial^2 \Pi^f_i}{\partial r \partial p} & \frac{\partial^2 \Pi^f_i}{\partial r^2} & \frac{\partial^2 \Pi^f_i}{\partial r \partial m} \\
\frac{\partial^2 \Pi^f_i}{\partial m \partial p} & \frac{\partial^2 \Pi^f_i}{\partial m \partial r} & \frac{\partial^2 \Pi^f_i}{\partial m^2}
\end{array}\right]
\]

\[
= \left[\begin{array}{ccc}
-\frac{(\gamma - 1)^{\gamma - 2}}{\gamma(c_m + c_R)^{\gamma - 1}} & 0 & 0 \\
0 & -\frac{1 - \alpha}{\gamma r} & \frac{a}{r} \\
0 & \frac{a}{r} & -\frac{1 - \beta}{m}
\end{array}\right]
\]

Part (II) can be proved by considering EQ (1) and EQ (2).

According to part (I), if \(0 < \gamma < 1\), then the price has a negative value, and will lead to negative profit. So, we focus only on situation where \(\gamma > 1\). Furthermore, according to part (II), it is necessary to \(1 - \alpha - \beta\) to have a positive value.

**Stackelberg policy**

In the Stackelberg (Stag) approach, players are classified as leader and follower. The leader chooses his strategy first, and then the follower observes this decision and makes his own strategy. It is necessary to assume that each enterprise is not willing to deviate from maximizing
his profit. In other words, each player chooses his best strategy. Here, the manufacturer is the leader, and the retailer is the follower. The manufacturer determines his wholesale price and advertising investment and acts as a leader by announcing it to the retailer in advance, and the retailer acts as a follower by choosing his retail price and advertising investment based on the manufacturer strategy. We denote Stag solution by \((w_S, p_S, m_S, r_S)\). The objective functions for the retailer and manufacturer are as below:

\[
\Pi^S_R = (p - w - c_R)D(r, m, p) - r,
\]
\[
\Pi^S_M = (w - c_M)D(r, m, p) - m;
\]
\[
D(r, m, p) = Kr^m m^p p^{-\gamma}
\]

Proposition 2

(I) The optimal decisions in Stag policy are:

\[
w_S = \frac{1 - \alpha}{\gamma - \alpha}c_R + \frac{\gamma - \alpha}{\gamma - 1}m
\]
\[
p_S = \frac{\gamma - \alpha}{\gamma - 1} \times \frac{r}{m}(c_R + c_M)
\]
\[
m_S = \frac{1 - \alpha}{\gamma - \alpha}k \alpha^2 \beta^{1 - \alpha} \frac{(y - 1)\gamma - \alpha - 1}{\sqrt{(c_R + c_M)^{\gamma - 1}} \times \frac{\gamma - \alpha}{\gamma}(y - \alpha) - \alpha}
\]
\[
r_S = \frac{1 - \alpha}{\gamma - \alpha}k \alpha^2 \beta^{1 - \alpha} \frac{(y - 1)\gamma - \alpha - 2}{\sqrt{(c_R + c_M)^{\gamma - 1}} \times \frac{\gamma - \alpha}{\gamma}(y - \alpha)^{\gamma - 2}}
\]

Proof: The first order conditions for the retailer are as follows by considering \(\partial D/\partial p = -\gamma D(r, m, p)/p\) and \(\partial D/\partial r = aD(r, m, p)/r\):

\[
\frac{\partial \Pi^S_R}{\partial p} = D(r, m, p) \left[ \frac{1 - \gamma}{\gamma} (p - w - c_R) \right] = 0
\]
\[
\frac{\partial \Pi^S_R}{\partial r} = D(r, m, p) (p - w - c_R) - 1 = 0
\]

Solving the above equations, we get the optimal solution for given \(w\) and \(m\) as follows:

\[
p = \frac{\gamma (w + c_R)(y - 1)}{(\gamma - 1)\gamma}
\]
\[
r = \frac{\gamma^2}{(\gamma - 1)\gamma^2} \times \frac{(w + c_R)^{\gamma - 1}}{k\alpha m^\gamma}
\]

Bear in mind that the Hessian matrix is a negative definite matrix, so \(\Pi^S_R\) is concave in \(p\) and \(r\), and have a maximum value.

\[
\begin{bmatrix}
\frac{\partial^2 \Pi^S_R}{\partial p^2} & \frac{\partial^2 \Pi^S_R}{\partial p \partial r}
\end{bmatrix} = \begin{bmatrix}
\frac{-(\gamma - 1)^2}{\gamma} & 0
\end{bmatrix}
\]

(7)

By substituting obtained values for \(p\) and \(r\) in the manufacturer’s function, we get the following equation which is a function of only \(w\) and \(m\):

\[
\Pi^S_M = (w - c_M) \times (w + c_R)^{\gamma - \alpha} \times m^{\gamma - 1}
\]
\[
\times \frac{1 - \alpha}{\gamma - \alpha} \times \frac{r}{m} \times \frac{1}{\gamma - 1} - m^{\gamma - \alpha}
\]

Then, the first order conditions for the manufacturer are:

\[
\frac{\partial \Pi^S_M}{\partial w} = D(r, m, p) \left[ 1 + \frac{w - c_M}{w + c_R} \frac{y - \alpha}{\alpha - 1} \right] = 0
\]
\[
\frac{\partial \Pi^S_M}{\partial m} = (w - c_M) \frac{\beta}{m} \frac{D(r, m, p)}{1 - \alpha} = 0
\]

(9)

Solving the above equations together with EQ (6), part (I) will be proved. Similarly, \(\Pi^S_M\) is concave in \(w\) and \(m\), because the Hessian matrix is a negative definite matrix.

\[
H = \begin{bmatrix}
\frac{\partial^2 \Pi^S_M}{\partial w^2} & \frac{\partial^2 \Pi^S_M}{\partial w \partial m} \\
\frac{\partial^2 \Pi^S_M}{\partial m \partial w} & \frac{\partial^2 \Pi^S_M}{\partial m^2}
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\beta(y - \alpha)(1 - \alpha)(c_M + c_R)^{2} & 0 \\
0 & -1 - \alpha - \beta \frac{1}{m(1 - \alpha)}
\end{bmatrix}
\]

Also, Part (II) can be proved in the same way as we proved in proposition 1.

Retail Fixed Markup (RFM) policy

In this policy, the manufacturer sets his wholesale price first. Next the retailer chooses his advertising investment. Here, similar to Stackelberg policy the manufacturer sets his decisions by considering that the retailer will choose his best strategy. The retailer receives a fixed markup \((\theta = 1 - w/p)\). So, choosing the wholesale price by manufacturer is equivalent to setting the retail price. We denote integrated solution by \((w_F, p_F, m_F, r_F)\). By Substituting \(w = (1 - \theta)p\) to EQ (4), we get the objective functions of two firms under RFM as below:

\[
\Pi^F_R = [(1 - \theta)p - c_M]D(r, m, p) - m
\]
\[
\Pi^F_M = \frac{\partial \Pi^F_R}{\partial r}D(r, m, p) - r;
\]
\[
D(r, m, p) = Kr^m m^p p^{-\gamma}
\]

Proposition 3

(I) The optimal retail price is one of the roots of the following equation:

\[
\theta(1 - \theta)(y - 1)p^2 + \gamma c_G c_M + [(1 - \alpha - \gamma)(1 - \theta)c_R - \theta(\gamma - \alpha)c_M]p_F = 0
\]

(II) Other optimal variables are:

\[
w_F = (1 - \theta)p_F
\]
\[
m_F = \frac{1 - \alpha}{\alpha} \times \frac{k \alpha^2 \beta^{1 - \alpha}}{(1 - \alpha - \theta)c_R - \theta(\gamma - \alpha)c_M} \times \frac{(\theta p_F - c_R)^{1 - \gamma}}{p_F^{1 - \gamma}}
\]

(III) The two firms’ profit at RFM policy is:

\[
\Pi^F_R = \frac{1 - \alpha}{\alpha} \times \frac{k \alpha^2 \beta^{1 - \alpha}}{(1 - \alpha - \theta)c_R - \theta(\gamma - \alpha)c_M} \times \frac{(\theta p_F - c_R)^{1 - \gamma}}{p_F^{1 - \gamma}}
\]

Proof: The retailer’s investment for given \(m\) and \(p\) is obtained from:

\[
\frac{\partial \Pi^F_R}{\partial r} = (\theta p - c_R)D(r, m, p) - 1 = 0
\]

(12)
By substituting obtained value for \( r \) in the manufacturer’s function and differentiating it with respect to \( p \) and \( m \), we get the following two equations:

\[
D(r, m, p) = \left\{ 1 - \theta + \frac{[1 - \theta]p - c_M}{a - 1} \left[ \frac{\theta a}{\beta p - c_M} - \frac{\theta a}{\beta p - c_M} \right] \right\} = 0
\]

\[
[(1 - \theta)p - c_M] \left[ \frac{\theta a}{\beta p - c_M} - 1 \right] - 1 = 0 \tag{13}
\]

Solving the above equations results part (I) and (II). Furthermore, Part (III) can be proved in the same way as we proved in proposition 1.

**Numerical study**

We perform a numerical study to quantify our analytical results and concepts from the previous sections to achieve some managerial insights. We present a base case to compare the results of different policies. Then, we illustrate the Pareto-improving region through a numerical study. Finally, we present a sensitivity analysis of results by changing the values of some parameters. We applied MAPLE 12 for evaluating the problem.

**Base case**

In our numerical study, we consider the same base-case values which used in YU et.al (2009). Values for input parameters presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter values of the base-case</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( c_M )</th>
<th>( c_R )</th>
<th>( \kappa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.43</td>
<td>0.39</td>
<td>1.3</td>
<td>20</td>
<td>20</td>
<td>350</td>
</tr>
</tbody>
</table>

Table 2 summarizes the solutions of three policies for the base-case values. In RFM policy it is assumed that \( \theta \) is equal to 0.54. As shown in the table, the retailer’s and manufacturer’s profit at RFM policy is higher than those of Stag policy. The manufacturer’s, retailer’s, and total supply chain’s profit increased by 1843%, 369%, and 516%, respectively at RFM policy compared to Stag policy.

**Table 2**

**Solutions of two approaches for the Base-case Example**

<table>
<thead>
<tr>
<th>( w^* )</th>
<th>Integrated</th>
<th>Stag</th>
<th>RFM(0.54)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^* )</td>
<td>173.3</td>
<td>96</td>
<td>81.3</td>
</tr>
<tr>
<td>( r^*(10^3) )</td>
<td>438</td>
<td>7.4</td>
<td>34.6</td>
</tr>
<tr>
<td>( m^*(10^3) )</td>
<td>397</td>
<td>2.3</td>
<td>44.8</td>
</tr>
<tr>
<td>( \Pi_M(10^3) )</td>
<td>-</td>
<td>1.06</td>
<td>21.6</td>
</tr>
<tr>
<td>( \Pi_R(10^3) )</td>
<td>18.3</td>
<td>9.79</td>
<td>46.9</td>
</tr>
<tr>
<td>( \Pi_T(10^3) )</td>
<td>1.08</td>
<td>6.66</td>
<td></td>
</tr>
</tbody>
</table>

**Lemma 1**: the manufacturer’s profit (\( \Pi_M^F \)) and the retailer’s profit (\( \Pi_R^F \)) are concave in \( \theta \) and have a maximum value.

**Proof**: Due to complexity of the problem, we show the concavity of the profit functions numerically. Figure 1 illustrate the two firm’s profit respect to \( \theta \). There exist \( \theta_1 \), and \( \theta_2 \) such that maximize the retailer’s and manufacturer’s profit, respectively. As shown in the figure, both the profit functions are concave. We know that sum of two concave function will be concave. Consider that \( \theta_1 = 0.58 \) and \( \theta_2 = 0.45 \), we can say that the value of \( \theta \) that maximize whole system’s profit at RFM policy is between 0.45 and 0.58. Here, when\( \theta = 0.54 \), the total channel’s profit is maximized.

**Figure 1.** Two firms’ and supply chain’s profit in \( \theta \)

**Figure 2** illustrates the retailer’s and manufacturer’s profit functions with respect to \( \theta \) under the RFM and Stackelberg policies. As shown in the figure, the retailer will prefer RFM policy to Stag policy if \( \theta \) is between 0.31 and 0.82, similarly the manufacturer will benefit from RFM policy if \( \theta \) is between 0.12 and 0.81. So, \( \theta \in (0.31, 0.81) \) is a Pareto-Efficient strategy.

**Figure 2.** Two firms’ profit at Stag and RFM policies respect to \( \theta \)

**Lemma 2**: RFM policy with \( \theta \in (\theta_1, \theta_2) \) leads to Pareto improvement over the Stag policy if \( \theta \) satisfies the both following constraints:

\[
r_F \geq r_S, \quad \frac{(1 - \theta)p - c_M}{(\theta p - c_R)(1 - a)} r_F \geq \frac{\gamma - 1}{\gamma - a} r_S \tag{14}
\]

We investigate this Lemma numerically. RFM (\( \theta \)) will be a Pareto-efficient strategy if both the retailer and manufacturer can benefit from it comparing to Stag policy, in other words, we should have \( \Pi_R^F \geq \Pi_R^S \) and \( \Pi_M^F \geq \Pi_M^S \)
which are equivalent to EQ (14). It can be proved easily from simultaneously using part (II) of proposition (II) and part (III) of proposition (III). From Lemma 2, for base case data \( \theta \in (0.31, 0.81) \) is a Pareto-efficient strategy. We investigate this interval numerically. We found that, the interval \( (\theta, \theta) \) is absolutely less sensitive to \( \alpha \) and \( k \) for \( \alpha \in (0.35, 0.55) \) and \( k \in (300, 400) \). Figure 3 and 4 illustrate the variations of this region with respect to \( \gamma \) and \( c_R \), respectively.

Sensitivity analysis

In this subsection, we perform a sensitivity analysis by changing the values of major parameters in the base-case. We define \( \rho \) as the supply chain’s percentage improvement at RFM policy comparing to Stag policy. Table 3 shows the Sensitivity analysis of total supply chain profit at Stag and RFM policies and percentage improvement of RFM policy. Consider that \( \theta^* \) at RFM policy maximize the total supply chain’s profit. Also, Table 4 shows the variations of decision variables at three policies.

![Figure 3. Pareto-improving region in \( \gamma \)](image)

![Figure 4. Pareto-improving region in \( c_R \)](image)

<table>
<thead>
<tr>
<th>Solutions ( \rightarrow ) Parameters ( \downarrow )</th>
<th>Stag</th>
<th>RFM ( (\theta^*) )</th>
<th>Stag</th>
<th>RFM ( (\theta^*) )</th>
<th>Stag</th>
<th>RFM ( (\theta^*) )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( 0.35 )</td>
<td>173</td>
<td>1.2</td>
<td>1.34</td>
<td>107</td>
<td>549</td>
<td>5.6</td>
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<tr>
<td>( 0.45 )</td>
<td>123</td>
<td>4690</td>
<td>4070</td>
<td>93</td>
<td>491</td>
<td>5260</td>
<td>1600</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 0.3 )</td>
<td>173</td>
<td>0.96</td>
<td>0.67</td>
<td>96</td>
<td>503</td>
<td>9</td>
</tr>
<tr>
<td>( 0.4 )</td>
<td>173</td>
<td>1300</td>
<td>1210</td>
<td>96</td>
<td>503</td>
<td>1620</td>
<td>520</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>( 1.2 )</td>
<td>240</td>
<td>8330</td>
<td>7560</td>
<td>134</td>
<td>924</td>
<td>10000</td>
</tr>
<tr>
<td>( 1.5 )</td>
<td>120</td>
<td>178</td>
<td>1.62</td>
<td>66</td>
<td>256</td>
<td>4.1</td>
<td>1.7</td>
</tr>
<tr>
<td>( k )</td>
<td>( 300 )</td>
<td>173</td>
<td>186</td>
<td>168</td>
<td>96</td>
<td>503</td>
<td>314</td>
</tr>
<tr>
<td>( 400 )</td>
<td>173</td>
<td>920</td>
<td>833</td>
<td>96</td>
<td>503</td>
<td>1550</td>
<td>485</td>
</tr>
<tr>
<td>( c_R )</td>
<td>( 50 )</td>
<td>303</td>
<td>170</td>
<td>156</td>
<td>153</td>
<td>880</td>
<td>290</td>
</tr>
<tr>
<td>( c_M )</td>
<td>( 5 )</td>
<td>108</td>
<td>957</td>
<td>868</td>
<td>33</td>
<td>314</td>
<td>1600</td>
</tr>
<tr>
<td>( 50 )</td>
<td>303</td>
<td>172</td>
<td>156</td>
<td>183</td>
<td>880</td>
<td>290</td>
<td>91</td>
</tr>
</tbody>
</table>

Furthermore, we solve 1000 problems and drive conclusions about the results. To evaluate the RFM policy, we set \( \theta^* \), and other parameters generated randomly as: \( \alpha \in (0.38, 0.45), \beta \in (0.38, 0.45), \gamma \in (1.2, 1.4), k \in (300, 400), c_R \in (10, 30), c_M \in (30, 50) \).

Solving the problems, we found that both the retailer and manufacturer can benefit from RFM \( (\theta^*) \) comparing to Stag policy in all problems. Then, we examine the percentage improvement of this policy comparing to Stag policy. Table 5 shows the results. The manufacturer’s and retailer’s profit increased at least by 900% and 156% respectively. Also, The Stag policy’s average efficiency is about 6% of that of integrated policy, while increased to 37% at RFM \( (\theta^*) \) policy.

Figures 5, 6, 7 and 8 illustrate the influence of \( \alpha \) or \( \gamma \) on prices, demand and total profit. We set \( \theta = \theta^* \) at RFM policy in which the total supply chain’s profit maximized. It is obvious that the retail price at Stag policy is much greater than those of other policies, so, much more demand will be lost in this policy because of higher retail price. The retail and wholesale prices are decreasing in \( \gamma \), while, they are not considerably changed by changing the value of \( \alpha \).

<table>
<thead>
<tr>
<th>Table 5</th>
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</table>

Percentage improvement of RFM comparing to Stag policy

<table>
<thead>
<tr>
<th></th>
<th>Manufacture</th>
<th>Retailer</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>2395</td>
<td>440</td>
<td>615</td>
</tr>
<tr>
<td>Minimum</td>
<td>900</td>
<td>156</td>
<td>252</td>
</tr>
<tr>
<td>Maximum</td>
<td>11370</td>
<td>1870</td>
<td>2420</td>
</tr>
<tr>
<td>Mode</td>
<td>2135</td>
<td>286</td>
<td>567</td>
</tr>
</tbody>
</table>

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Conclusions

This paper has studied the retailer-manufacturer problem in which the market demand is a decreasing and convex function of the retail price, but an increasing and concave function of advertising investment of two firms. Both firms are willing to optimize their profit by adjusting pricing and advertising decisions. We compare the problem under integrated and Stackelberg policies. Then, we examine Retail Fixed Markup (RFM) policy and investigate its performance on supply chain.

We found numerically that: (a) a properly designed RFM improves each member’s profit and leads to Pareto improvement over Stackelberg policy. The manufacturer’s and retailer’s profit increased at least by 900% and 156% respectively comparing to Stackelberg policy. Besides, it improves the total supply chain’s profit by 600% in average comparing to Stackelberg policy. (b) Both the manufacturer’s and the retailer’s profit are concave in

Retail Fixed Markup rate and have a maximum value. Consequently, the total channel’s profit is concave and has a maximum. (c) The Pareto-improving region is less sensitive to some parameters such as investment’s elasticity and scale k, while other parameters such as price’s elasticity, production and distribution costs have considerable effect on the region. (d) RFM policy’s average efficiency is 37% of that of integrated policy. So, there is an opportunity to achieve higher channel efficiency than this.

Our research has some limitations for future research. We consider a channel with one manufacturer and a single retailer that is not in accordance with real world supply chains. So, a more general model with multiple retailers or multiple manufacturers will produce interesting results. In addition, one can adopt a more general demand function to investigate the problem.
References


Pastaraisiais metais buvo atliktu daug tyrimų, susijusių su įvairiais tiekimo grandinės aspektais: kainų nustatymu, rinkodara, gamybos, paskirstymu, prikimui, turto valdymu, ir t. t. Mažmenininko ir gamintojo tarpusavio sąveikos problema yra viana iš klasikinių tyrimo srčių, kuri yra analizuojama mokslinguje literatūroje apie tiekimo grandinės. Siuos tyrimus siekiama atsakyti į keletą klausimų: “Kaip turėtų eiti tiekimo grandinės dalvysiai norėdami valdyti savo pelną?” ir “Kaip gamintojas gali panaudoti savo lyderystės privalumus, norėdamas padidinti savo ir mažmenininko pelną?”
